Effects of a Deterministic Hill Climber on hBOA

Elizabeth Radetic, Martin Pelikan, and David E. Goldberg

Missouri Estimation of Distribution Algorithms Laboratory (MEDAL)  
University of Missouri, St. Louis, MO  
http://medal.cs.umsl.edu/  
err26c@umsl.edu  pelikan@cs.umsl.edu

Illinois Genetic Algorithms Laboratory  
University of Illinois at Urbana-Champaign, Urbana, IL  
http://www-illigal.ge.uiuc.edu/ deg@uiuc.edu
Hybrid Estimation of Distribution Algorithms (EDAs)

- For very large or complex problems, scalability may not be sufficient; additional efficiency enhancement may be required
- Hybridization is a popular method of efficiency enhancement for EDAs and other GEAs
- A hybrid combines a local search algorithm with a global searcher such as an EDA

Hybrid hBOA

- The hierarchical Bayesian optimization algorithm (hBOA) is frequently hybridized
- This study focuses on hBOA combined with a deterministic hill climber (DHC)
  - Examine the effects of varying the parameters of DHC
  - Study the effects of variance reduction on model building
Outline

1. Algorithms
2. Goals & Methodology
3. Test Problems
4. Results
5. Discussion of Results
6. Summary and conclusions.
Hierarchical Bayesian Optimization Algorithm (hBOA)

- Randomly initialize a population of binary strings
- Select promising solutions from the population
- Build a Bayesian network from the selected solutions
- Generate new solutions from the constructed model
- Incorporate new solutions into the population using restricted tournament replacement
Deterministic Hill Climber (DHC)

- Flip one bit in the string that will produce the most fitness improvement
- Repeat until a specified quality has been reached, a maximum number of flips has been performed, or no more improvement is possible
- hBOA has frequently been combined with DHC to improve performance (e.g., Pelikan & Goldberg, 2003; Pelikan & Hartmann, 2006; Pelikan, Katzgraber, & Kobe, 2008)
Experiment Goals

- Examine the effects of DHC on performance of hBOA
- Determine optimal settings for DHC

Methodology

- Tune the frequency of local search, i.e. the proportion of strings in the population on which DHC operates
- Tune the duration of local search, i.e. the maximum number of flips per string
- Compare
  - Population size
  - Number of generations until optimum
Test Problems

- Trap-5
- 2-D Ising Spin Glass
- 3-CNF MAXSAT
Test Problems

Trap-5

- Concatenation of non-overlapping subproblems of 5 bits
- The fitness of each subproblem is determined as

\[ f_{\text{trap}_5}(u) = \begin{cases} 
5 & \text{if } u = 5, \\
4 - u & \text{otherwise}
\end{cases} \]

where \( u \) is the number of ones in the subproblem.
- All subproblems are added together to determine the string’s fitness.
Test Problems

Trap-5

- Properties
  - Separable, non-overlapping subproblems
  - Subproblems are fully deceptive
  - Difficult for local search, simple GA

- Parameters
  - Problem sizes of 100-350 bits
  - Population size determined empirically using bisection method
    - 10 independent bisections of 10 runs each
2-D Ising Spin Glass

- Spins are arranged on a 2-D grid
- Each spin $s_i$ gets a value from $+1, -1$
- Each edge between neighboring spins $s_i$ and $s_j$ is assigned a real value $J_{i,j}$
- The task is to find a configuration of spins to minimize the energy, specified as

$$H(s) = - \sum_{i,j=0}^{n} J_{i,j} s_i s_j$$

- Spins are represented as bits in the binary string, 1 for $+1$, 0 for $-1$
2-D Ising Spin Glass

- **Properties**
  - More complex fitness landscape than trap
  - Not fully decomposable into subproblems of bounded order, substructures overlap
  - Solvable in polynomial time

- **Parameters**
  - Problem sizes 8x8, 10x10, 12x12, 14x14, 16x16 (64 - 256 bits)
  - 1000 instances
    - For each instance, $J_{ij}$ randomly set to $+1$ or $-1$
  - Population size determined using bisection method, 10 successful runs per instance
MAXSAT

- Instances consist of 3-CNF formulas: conjunctions of disjunctions of 3 literals
- The task is to find an assignment of Boolean variables to satisfy the maximum number of clauses
- Bits in the string represent the variables, with 1 representing true and 0 representing false
Test Problems

MAXSAT

- Properties
  - Complex fitness landscape
  - Not fully decomposable into subproblems of bounded order, substructures overlap
  - Not solvable in polynomial time

- Parameters
  - Problem sizes of 50 and 75 variables
  - 100 randomly-generated instances
  - Population size determined using bisection method, 10 successful runs per instance

Elizabeth Radetic, Martin Pelikan, and David E. Goldberg
Effects of a Deterministic Hill Climber on hBOA
Experimental Results

Trap-5 (350 bits)

- Reduction factor, speed-up: ratio of values without DHC to those with DHC

![Graph showing population size reduction factor and generations speed-up for different max. flips (% of prob. size) and p_dhc values ranging from 0.1 to 0.9.](image)
Experimental Results

Trap-5 (350 bits)

![Graph showing evaluations speed-up and flips per evaluation for different values of \( p_{dhc} \).]
Experimental Results

Trap-5

Elizabeth Radetic, Martin Pelikan, and David E. Goldberg

Effects of a Deterministic Hill Climber on hBOA
Experimental Results

Ising Spin Glass (16x16, 256 bits)

Elizabeth Radetic, Martin Pelikan, and David E. Goldberg

Effects of a Deterministic Hill Climber on hBOA
Experimental Results

Ising Spin Glass (16x16, 256 bits)
Experimental Results

Ising Spin Glass

Elizabeth Radetic, Martin Pelikan, and David E. Goldberg
Effects of a Deterministic Hill Climber on hBOA
Experimental Results

MAXSAT (75 bits)

Max. Flips (% of Prob. Size) vs. Population Reduction Factor (original/reduced)

Max. Flips (% of Prob. Size) vs. Generation Speed-up

Elizabeth Radetic, Martin Pelikan, and David E. Goldberg

Effects of a Deterministic Hill Climber on hBOA
Experimental Results

MAXSAT (75 bits)

Evaluation speedups

Flips per Evaluation

Max. Flips (% of Prob. Size)

$\text{p}_{\text{dhc}}$

0.1
0.2
0.3
0.4
0.5
0.6
0.7
0.8
0.9
1.0

Elizabeth Radetic, Martin Pelikan, and David E. Goldberg

Effects of a Deterministic Hill Climber on hBOA
Discussion of Results

Effects of DHC

- DHC improved performance on each test problem, for all sizes
- **Substantial benefits were obtained despite the fact that DHC by itself performs poorly on each of these problems**
- Benefits of DHC are mainly due to variance reduction
  - Reduced variance makes model building easier
  - Also makes decision making easier
Discussion of Results

Effects of DHC on Model Building in hBOA: Trap-5

- Model quality
  - Without DHC, the perfect model for trap-5 requires connections between each variable within the partition, 10 links in total
  - With DHC, each partition contains either 11111 or 00000, so fewer dependencies (4) are required
Discussion of Results

Effects of DHC on Model Building in hBOA: Trap-5 (200 bits)

- Model quality: Proportion of correct dependencies
  - Without DHC, much larger populations are required to obtain the 10 necessary dependencies
  - With DHC, smaller populations are required to find the 4 necessary dependencies

First generation

Middle of the run
Discussion of Results

Effects of DHC on Model Building in hBOA: Trap-5 (200 bits)

- Model quality: proportion of incorrect dependencies
  - Without DHC, very many incorrect (spurious) dependencies are discovered for all but very large populations
  - With DHC, very few spurious dependencies are discovered, except for very large populations

First generation

Middle of the run
DHC’s Effects on hBOA

▸ DHC significantly improves the performance of hBOA on each of the test problems

▸ In these experiments the maximum improvement was attained by allowing DHC to operate on the full population and run until it found the local optimum

▸ DHC helps by reducing the variance
  ▸ Model building is easier
  ▸ Decision making is also easier
Acknowledgments

- NSF; NSF CAREER grant ECS-0547013.
- U.S. Air Force, AFOSR; FA9550-06-1-0096.
- University of Missouri; High Performance Computing Collaboratory sponsored by Information Technology Services; Research Award; Research Board.