Analyzing Probabilistic Models in Hierarchical BOA on Traps and Spin Glasses

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Motivation

- Background
  - hBOA can solve difficult problems scalably and reliably
  - Much empirical work has been done
    - efficiency enhancement (EE)
    - scalability
  - Relatively little studying of the models themselves

- Purpose
  - Understand models in hBOA
    - Design problem-specific EE
    - Design automated EE techniques
    - Educate researchers about their problems
Outline

- Hierarchical BOA (hBOA)
- Models in hBOA
- Experiments
  - Trap 5
  - 2d Ising Spin Glass
- Conclusions
- Future Work
Hierarchical Bayesian Optimization Algorithm (hBOA)

- Pelikan, Goldberg and Cantu-Paz (2001)
- Uses Bayesian network with local structures to model promising solutions
  - Bayesian network
    - Acyclic directed Graph
    - String positions are the nodes
    - Edges represent conditional dependencies
    - Where there is no edge, implicit independence
  - Sampled to create new population each gen.
- Niching to maintain diversity
hBOA

Current population

Selection

Bayesian network

New population
Bayesian Network

- **Structure**
  - Edges determine dependence

- **Parameters**
  - \( p(\text{Burglary}|\text{Alarm}) \)
What are we looking for?

- **Accuracy**
  - Do the models accurately represent the underlying problem structure
    - Effects on scalability
    - Using models to understand problems

- **Dynamics**
  - How the models change over time
    - Efficiency Enhancement
Test Problems

- Selecting our test problems
  - Have to know what a good model looks like
  - Non-trivial
  - Easily scaled up
  - Diverse

- Test Problems Selected
  - Trap-5
  - 2D Ising Spin Glass
Trap-5

- Partition binary string into disjoint groups of 5 bits

\[ \text{trap}(\text{ones}) = \begin{cases} 
5 & \text{if } \text{ones} = 5 \\
4 - \text{ones} & \text{otherwise}
\end{cases} \]

- Total fitness is sum of single traps
- Optimum: String 1111...1
Perfect Model for Trap-5

- Each 5-bit partition should be fully (or close to fully) connected (10 edges)
  - Necessary dependencies
- Bits between partitions should be independent
  - Unnecessary dependencies
Trap-5 Experiments

- Problem sizes 15-210
- 30 runs for each size
- Bisection was used to determine population size
Trap-5 Results

- Necessary and unnecessary dependencies w.r.t problem size
- Three snapshots taken in each run (first, middle, last gen.)
- Average of the 30 runs for each problem size
Trap-5 Results

- Ratio of unnecessary dependencies to total dependencies as problem size increases

a) Middle snapshot

b) End snapshot
Trap-5 Results

- Model dynamics for two different problem sizes.
- Dependencies were counted as the same even if their direction changed.

a) $N = 100$ bits.

b) $N = 200$ bits.
2D Ising Spin Glass

- Origin in physics
- Spins arranged on a 2D grid (periodic boundary conditions)

- Each spin ($s_j$) can have two values: +1 or -1
- Set of connection weights $J_{ij}$ specifies an instance.

\[
E = E(c) - \sum_{i,j} s_i J_{i,j} s_j
\]
2D Ising Spin Glass

- Very hard for most optimization techniques
  - Extremely large number of local optima
  - Any decomposition of bounded order is insufficient
  - The problem is solvable in polynomial time by analytical techniques

- hBOA does solve it in polynomial time
  - A deterministic hill-climber (DHC) is used to improve the quality of all evaluated solutions
    - Single bit flips until no improvements
Perfect Model for Spin-Glass

- Not sure what the perfect model is
- To some degree, all positions are dependent on all others
  - If we consider all, it will not scale
- We would expect the interactions should decrease in magnitude as their distance increases
Spin Glass Experiments

- 100 different instances of size 10x10 to 20x20
  - Only graphs of 20x20 are shown
  - With and without DHC
  - 5 runs of each instance

- Scaling as we restrict models
  - Restrict models to short order dependencies
Spin Glass Results

a) First generation

b) Second snapshot

c) Third snapshot

d) Last generation
Spin Glass Results (No DHC)

a) First generation

b) Second snapshot

c) Third snapshot

d) Last generation
Spin Glass Results

- One example instance (rest similar)
- Dependencies were counted the same even if their direction changed

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**a) All dependencies**

- Total
- + New Dep
- − Old Dep

**b) Neighbor dependencies**

- Total
- + New Dep
- − Old Dep
Restricting hBOA models

- Evaluations varying by different restrictions
- Experiments without DHC had similar scaling
Conclusions

- Studying hBOA models is challenging but possible
- hBOA generates accurate models quickly
- Models reveal a lot of information about the problem
- Models do not change rapidly over time
- Information about models can help us design EE
Future Work

- Do model analysis on other challenging problems
  - Artificial
  - Real world
- Develop techniques to automatically exploit this information for EE
Any Questions?