Obtaining Ground States of Ising Spin Glasses via Optimizing Bonds Instead of Spins

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Motivation
• Ising spin glasses are prototypical models for disordered systems.
• Ising spin glasses are also challenging class of optimization problems
  • Fast growth of the number of local optima.
  • High-quality solutions divided by barriers of low-quality ones.
  • High-order interactions.
  • Cannot be factored into subproblems of bounded order.
• Hierarchical BOA (hBOA) performs very well on Ising spin glasses and other difficult combinatorial and constraint satisfaction problems.
• However, standard GAs and local search perform poorly.
• Questions
  • Can we improve performance of standard evolutionary algorithms on spin glasses by transforming the problem in some way?
  • Can we generalize this problem transformation to apply to other important classes of constraint satisfaction problems (CSPs), such as MAXSAT?

Goals
• Transform the Ising spin glass problem to optimize bonds instead of spins in order to simplify the problem.
• Test various evolutionary algorithms on the transformed problem and analyze the effects of the transformation.

Ising spin glass
• Ising spin glass
  • Spins arranged on a 2D or 3D grid.
  • Spins \{s_i\} can obtain values 2 values: +1 or -1.
  • Neighbors connected (+ periodic boundary conditions)
  • Each connection (i,j) has a weight \(J_{i,j}\)
  • Spin glass instance specified by all \(J_{i,j}\)

• Energy of a spin glass
  \[ E(C) = \sum_{(i,j)} s_i J_{i,j} s_j \]
• Optimization problem: Find ground state
  • Given all coupling constants \(J_{i,j}\)
  • Find values of spins so that energy is minimized.

Optimizing bonds instead of spins
• Candidate solutions represented by binary vectors (1 bit = 1 spin).
• After selection, spin vectors are transformed into bond vectors (another representation).
• Variation is applied to the transformed solutions.
• The new solutions are transformed back into spin vectors.

Transforming spins to bonds and vice versa
• Spins to couplings (S \(\rightarrow\) C)
  • Each coupling maps to a bit.
  • 1: Satisfied constraint (negative energy)
  • 0: Unsatisfied constraint (positive energy)
• Couplings to spins (C \(\rightarrow\) S)
  • Start in a random spin, set it to random value.
  • In each step choose a spin to set to maximize the number of consistent couplings (ties are resolved randomly).

Experiments
• Test problems
  • 2D spin glasses of size 6x6 to 16x16.
  • 1000 random instances for each size (couplings are +1 or -1)

Genetic algorithm (two-point crossover)

Univariate marginal distribution algorithm (UMDA)

Hierarchical BOA (hBOA)

Conclusions
• Transformation significantly helps simple evolutionary algorithms like GA and UMDA.
• However, hBOA works better without transformation.
• Can we generalize these results?

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